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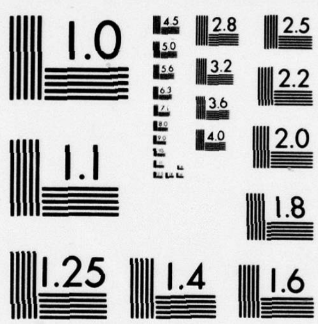

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NOTE ON A NON-TRIVIAL SIMPLE EXAMPLE OF HIGHER-ORDER ONE-DIMENSIONAL BEAM THEORY

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by E. Reissner

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by

E. Reissner

ABSTRACT

One-dimensional beam equations are derived for a rectangular-cross section beam consisting of shear webs and corner stringers with one cross-sectional axis of symmetry. It is shown that a systematic statement of this problem involves one more equation than elementary theory, with this additional equation involving the concept of cross-sectional warping and bi-moment. As an application of the general results of the paper, new conclusions are deduced concerning the location of the center of shear of the beam.

Note on a Non-Trivial Simple Example of Higher-Order

One-Dimensional Beam Theory†

by

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Introduction. We depart from the well-known fact that elementary one-dimensional theory of space-curved beams involves a system of six equilibrium equations, six strain displacement equations and six constitutive equations for forces and moments, translational and rotational displacements, and force and moment strains, and that this theory may turn out to be inadequate, for beams with properties other than those corresponding to compact and reasonably homogeneous cross sections. In order to gain insight into situations which may be encountered, we consider what we think is the simplest non-trivial example of a problem of this nature, to wit the problem of deriving the equations of (linear) one-dimensional beam theory for a straight beam having a rectangular cross section, with one axis of symmetry, as indicated in Figure 1. We assume that the bending stiffness of this beam is concentrated in four corner stringers and that transverse shear and torsional stiffness are concentrated in four shear webs††. In order to include in our analysis the limiting case of torsion of a thin-walled open cross section beam, we assume that the four webs possess not only inplane (sheet) shear stiffness, but also transverse (plate) twisting stiffness.

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† A report on work supported by the Office of Naval Research.

†† Consideration of more general configurations involving stringers and shear webs, but from a somewhat different point of view may be found in [1].

The relevant equations of statics of elementary beam theory with which we are concerned in what follows are three equations for bending moment M, transverse force Q and twisting moment T, of the form $Q' + q = 0$, $M' - Q + m = 0$ and $T' + t = 0$. We will show that a rational treatment on the basis of an elementary consideration of the three-dimensional aspects of the problem, by necessity introduces a fourth equation of statics of the form $R' - P + r = 0$. In this equation the quantity R has the character of a one-dimensional axial warping stress measure which is effectively equivalent to what has been designated by Vlasov as "bi-moment," and the quantity P has the character of a one-dimensional cross sectional shear stress measure, other than Q and T, the significance of which becomes apparent in the course of the step from three-dimensional formulation to one-dimensional formulation.

Having four one-dimensional equilibrium equations for Q, M, T, P and R we use an appropriate form of the principle of virtual work for the derivation of strain displacement relations in a manner which is simple but not self-evident. Finally, we use an appropriate expression for a three-dimensional complementary energy density function in order to establish a system of one-dimensional constitutive equations.

As an application of our system of one-dimensional beam equations, we consider once more the problem of determining the shear center of the cross section of our beam, with this approach leading here to more detailed insights than are believed to have been known before.

Statics and Virtual Work. We introduce axial forces N_1, N_2, shear flows S_1, S_2, S_{12} and plate twisting moments M_1, M_2, M_{12} as indicated in Figure 2, together with load intensities n, s and m as also indicated in the Figure.

Denoting differentiation with respect to an axial coordinate z by primes, we then read from Figure 2 the following four equilibrium equations

$$N_2' - S_2 + S_{12} + n_2 = 0 \ , \quad N_1' + S_1 - S_{12} + n_1 = 0 \ , \qquad \text{(1a,b)}$$

$$2c(S_1 - S_2)' + 2c(s_1 - s_2) = 0 \ , \qquad \text{(1c)}$$

$$2c(S_2 + S_1)'a + 2aS_{12}'2c + 2c(s_2 + s_1)a + 2as_{12}2c$$

$$+ 2[2c(M_1 + M_2) + 4aM_{12}]' + 2c(m_1 + m_2) + 4am_{12} = 0 \ . \qquad \text{(1d)}$$

We note that the factor "2" in front of the M-terms in Eq. (1d) has been introduced in order to account for the effect of reactive plate transverse shear stress resultants in a way corresponding to the result obtained by an explicit treatment of the problem of torsion in thin-walled open and closed cross section beams [2].

For what follows it is appropriate to rewrite Eqs. (1) in terms of one-dimensional beam stress resultants and couples Q, M and T and in terms of supplementary one-dimensional stress measures P and R, in conjunction with analogously defined load intensities q, m, t and r. Furthermore, account is taken of the fact that the torque T is composed of three distinct portions T_A, T_B, T_C. The relevant defining relations are

$$2c(S_2 - S_1) = Q \ , \quad 2c(N_1 + N_2) = M \ , \qquad \text{(2a,b)}$$

$$4acS_{12} = T_A \ , \quad 2ca(S_1 + S_2) = T_C \ , \qquad \text{(2c,d)}$$

$$2\oint M_s ds = 4c(M_1 + M_2) + 8aM_{12} = T_B \ , \qquad \text{(2e)}$$

$$T = T_A + T_B + T_C \ , \quad P = T_C - T_A \ , \quad R = 2ca(N_2 - N_1) \ . \qquad \text{(2f,g,h)}$$

With these we obtain directly from (1c) and (1d) as two of three standard equations of one-dimensional beam theory

$$Q' + q = 0 \quad , \quad T' + t = 0 \quad . \tag{3a,b}$$

The third standard equation follows by adding Eq. (1a) to Eq. (1b), in the familiar form

$$M' - Q + m = 0 \quad .$$

Finally, the non-standard fourth equation, involving the bi-moment R and the shear stress measure P follow by <u>subtracting</u> Eq. (1b) from Eq. (1a), in the form

$$R' - P + r = 0 \quad , \tag{3d}$$

where use has been made of the defining relations (2c), (2d) and (2h).

We next derive a system of strain displacement relations which is consistent with the equilibrium relations (3) by writing as equation of virtual work

$$\int (q\delta v + m\delta\phi + t\delta\theta + r\delta\psi)dz$$

$$= \int (Q\delta\gamma + M\delta\kappa + T_A\delta\tau_A + T_B\delta\tau_B + T_C\delta\tau_C + R\delta\lambda)dz \quad . \tag{4}$$

Equation (4) leads to a system of virtual strain displacement relations upon eliminating in it the quantities q, m, t, r through use of Eqs. (3), and upon then considering the resulting equation as an identity in Q, M, T_A, T_B, T_C and R. In view of the linearity of the problem the step from virtual to actual strain displacement relations is immediate and we have as the desired system of relations

$$\gamma = v' + \phi \quad , \quad \kappa = \phi' \quad , \quad \tau_A = \theta' - \psi \quad , \tag{5a,b,c}$$

$$\tau_B = \theta' \quad , \quad \tau_C = \theta' + \psi \quad , \quad \lambda = \psi' \quad . \tag{5d,e,f}$$

Constitutive Relations. We depart from the complementary energy density expression

$$W = \frac{N_1^2}{A_1E_1} + \frac{N_2^2}{A_2E_2} + \frac{cS_1^2}{t_1G_1} + \frac{cS_2^2}{t_2G_2} + \frac{2aS_{12}^2}{t_{12}G_{12}} + 2\oint \frac{M_s^2 ds}{2D_s} \quad , \tag{6}$$

where $D_s = G_s t_s^3/6$. We rewrite the various terms in this expression in terms of one-dimensional beam theory stress measures through use of the defining relations (2), as follows.

$$\begin{aligned}\frac{N_1^2}{A_1E_1} + \frac{N_2^2}{A_2E_2} &= \frac{(M-R/a)^2}{16c^2A_1E_1} + \frac{(M+R/a)^2}{16c^2A_2E_2} \\ &= \frac{1}{16c^2}\left[\left(\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right)\left(M^2 + \frac{R^2}{a^2}\right) + 2\left(\frac{1}{A_2E_2} - \frac{1}{A_1E_1}\right)\frac{MR}{a}\right] \\ &\equiv \frac{M^2}{2D_M} + \frac{R^2}{2D_R} + \frac{MR}{D_{MR}} \quad ,\end{aligned} \tag{7}$$

$$\frac{2aS_{12}}{t_{12}G_{12}} = \frac{2aT_A^2}{16c^2a^2t_{12}G_{12}} \equiv \frac{T_A^2}{2D_A} \quad , \tag{8}$$

$$\begin{aligned}\frac{cS_1^2}{t_1G_1} + \frac{cS_2^2}{t_2G_2} &= \frac{(Q-T_C/a)^2}{16ct_1G_1} + \frac{(Q+T_C/a)^2}{16ct_2G_2} \\ &= \frac{1}{16c}\left[\left(\frac{1}{t_1G_1} + \frac{1}{t_2G_2}\right)\left(Q^2 + \frac{T_C^2}{a^2}\right) + 2\left(\frac{1}{t_2G_2} - \frac{1}{t_1G_1}\right)\frac{QT_C}{a}\right] \\ &\equiv \frac{Q^2}{2D_Q} + \frac{T_C^2}{2D_C} + \frac{QT_C}{D_{CQ}} \quad ,\end{aligned} \tag{9}$$

$$\begin{aligned}2\oint \frac{M_s^2}{2D_s} ds &= \oint D_s \tau_s^2 ds = \tau_B^2 \oint D_s ds = \frac{T_B^2}{D_B^2}\oint D_s ds \\ &\equiv \frac{T_B^2}{2D_B} \quad , \qquad D_B = 2\oint D_s ds \quad .\end{aligned} \tag{10}$$

The meaning of the stiffness coefficients D_M, D_R, D_{RM}, D_A, D_Q, D_C and D_{CQ} may be read directly from Eqs. (7) to (9). An expression for D_B follows upon introducing the expression for D_s in Eq. (6), in the form

$$D_B = \frac{2}{3}\left[(G_1 t_1^3 + G_2 t_2^2)c + 2G_{12}t_{12}^3 a\right] \quad . \tag{11}$$

Having Eqs. (6) to (10) we have further, as a consequence of Castigliano's theorem, as the appropriate system of constitutive relations

$$\gamma = \frac{\partial W}{\partial Q} = \frac{Q}{D_Q} + \frac{T_C}{D_{CQ}} \ , \quad \kappa = \frac{\partial W}{\partial M} = \frac{M}{D_M} + \frac{R}{D_{MR}} \ , \tag{12a,b}$$

$$\tau_C = \frac{\partial W}{\partial T_C} = \frac{T_C}{D_C} + \frac{Q}{D_{CQ}} \ , \quad \lambda = \frac{\partial W}{\partial R} = \frac{R}{D_R} + \frac{M}{D_{MR}} \ , \tag{12c,d}$$

$$\tau_A = \frac{\partial W}{\partial T_A} = \frac{T_A}{D_A} \ , \quad \tau_B = \frac{\partial W}{\partial T_B} = \frac{T_B}{D_B} \ . \tag{12e,f}$$

End-Loaded Cantilever. Determination of Center of Shear. We assume as boundary conditions for the loaded end of the beam

$$z = 0 \ ; \quad Q = Q_o \ , \quad T = T_o \ , \quad M = R = 0 \ , \tag{13}$$

and as conditions for the supported end

$$z = L \ ; \quad v = 0 \ , \quad \theta = 0 \ , \quad \phi = 0 \ , \quad \psi = 0 \ . \tag{14}$$

We solve the homogeneous equilibrium equations (3a,b,c), as in the elementary theory

$$Q = Q_o \ , \quad T = T_o \ , \quad M = zQ_o \ , \tag{15a,b,c}$$

and leave the homogeneous equation (3d) in the form $R' - T_C + T_A = 0$.

The constitutive equations (12) in conjunction with the strain displacement relations (5) are now

$$v' + \phi = \frac{Q_o}{D_Q} + \frac{T_C}{D_{CQ}} \ , \quad \phi' = \frac{zQ_o}{D_M} + \frac{R}{D_{MR}} \ , \tag{16a,b}$$

$$\theta' - \psi = \frac{T_C}{D_C} + \frac{Q_o}{D_{CQ}} \quad , \quad \psi' = \frac{R}{D_R} + \frac{zQ_o}{D_{MR}} \quad , \tag{16c,d}$$

$$\theta' = \frac{T_B}{D_B} \quad , \quad \theta' + \psi = \frac{T_A}{D_A} \quad , \tag{16e,f}$$

where, according to (2f) and (15b), $T_A + T_B + T_C = T_o$.

We next introduce R, T_C and T_A from (16) into the remaining equilibrium equation and obtain as one of two differential equations for ψ and θ,

$$D_R\psi'' - (D_A + D_C)\psi + (D_A - D_C)\theta' = \left(\frac{D_R}{D_{MR}} - \frac{D_C}{D_{CQ}}\right)Q_o \; . \tag{17a}$$

A second differential equation follows from (16) in conjunction with the condition of constant T in the form

$$(D_A + D_B + D_C)\theta' - (D_A - D_C)\psi = T_o + \frac{D_C}{D_{CQ}}Q_o \quad . \tag{17b}$$

The three boundary conditions for the third-order system (17) are seen to be, from (14) (13) and (16d)

$$\theta(L) = \psi(L) = 0 \quad , \quad \psi'(0) = 0 \quad . \tag{18}$$

With ψ and θ from (17) and (18) we may subsequently determine ϕ and v, on the basis of (16a,b) from the relations

$$\phi' = \frac{zQ_o}{D_M}\left(1 - \frac{D_R D_M}{D_{MR}^2}\right) + \frac{D_R\psi'}{D_{MR}} \quad , \tag{19a}$$

$$v' = -\phi + \frac{Q_o}{D_Q}\left(1 - \frac{D_C D_Q}{D_{CQ}^2}\right) + \frac{D_C(\theta' + \psi)}{D_{CQ}} \quad , \tag{19b}$$

in conjunction with the boundary conditions $\phi(L) = v(L) = 0$.

We will limit ourselves here to a determination of the functions ψ and θ. The value θ_o of θ at the loaded end z = 0, when written in the

form $\theta_o = C_{\theta Q}Q_o + C_{\theta T}T_o$, then gives the coordinate x_s of the shear center of the cross section in the form $-C_{\theta Q}/C_{\theta T}$ [3,4].

We find from Eqs. (17a,b), in conjunction with the second and third boundary condition in (18), as expression for ψ, with $D = D_A+D_B+D_C$,

$$\psi = \left[\frac{(D_A-D_C)T_o}{4D_AD_C+D_B(D_A+D_C)} - \left(\frac{D_R}{D_{RM}} - \frac{D_C}{D_{CQ}}\frac{2D_A+D_B}{D}\right)\frac{DQ_o}{4D_AD_C+D_B(D_A+D_C)}\right]\left(1 - \frac{\cosh \mu z}{\cosh \mu L}\right); \quad (20)$$

where

$$\mu^2 = [4D_AD_C + D_B(D_A + D_C)]/D_RD \quad ,$$

and then, writing (17b) with $\theta(L) = 0$ in the form,

$$D\theta = \int_L^z [(D_A - D_C)\psi + T_o + (D_C/D_{CQ})Q_o]dz \ ,$$

$$\begin{aligned}\frac{\theta}{L} = {} & \frac{D_A+D_C}{4D_AD_C+D_B(D_A+D_C)}\left[T_o - \left(\frac{D_A-D_C}{D_A+D_C}\frac{D_R}{D_{RM}} - \frac{2D_A}{D_A+D_C}\frac{D_C}{D_{CQ}}\right)Q_o\right]\left(\frac{z}{L} - 1\right) \\ & - \frac{D_A-D_C}{4D_AD_C+D_B(D_A+D_C)}\left[\frac{D_A-D_C}{D}T_o\right. \\ & \left. - \left(\frac{D_R}{D_{RM}} - \frac{2D_A+D_B}{D}\frac{D_C}{D_{CQ}}\right)Q_o\right]\frac{\sinh \mu z - \sinh \mu L}{\mu L \cosh \mu L} \ . \end{aligned} \quad (21)$$

Equation (21) as it stands makes it possible to assess the effect of finite axial length L on the value of the shear center coordinate. We will here not concern ourselves with this question and limit attention to cases for which μL is sufficiently large to allow omission of the terms with μ in the value of θ_o. We then get from Eq. (21) as expression for the location of the shear center

$$x_s = \frac{D_A-D_C}{D_A+D_C}\frac{D_R}{D_{RM}} - \frac{2D_A}{D_A+D_C}\frac{D_C}{D_{CQ}} \ . \quad (22)$$

Having (22) we may determine the distance of the shear center from the elastic centroid of the cross section, as follows. We have, if we designate the coordinate of this centroid by x_c, $E_2A_2(a - x_c) = E_1A_1(a + x_c)$ and therewith

$$x_c = a\frac{E_2A_2-E_1A_1}{E_2A_2+E_1A_1} = -\frac{D_R}{D_{RM}} . \tag{23}$$

With this we obtain then from Eq. (22)

$$x_s - x_c = \frac{2D_A}{D_A+D_C}\left(\frac{D_R}{D_{RM}} - \frac{D_C}{D_{CQ}}\right) , \tag{24}$$

where, from Eqs. (8) and (9),

$$\frac{D_C}{D_{CQ}} = -a\frac{t_2G_2-t_1G_1}{t_2G_2+t_1G_1} ; \tag{25}$$

and

$$\frac{D_A}{D_A+D_C} = \frac{1}{1+\frac{a}{c}\,\frac{2t_1G_1t_2G_2}{(t_1G_1+t_2G_2)t_{12}G_{12}}} \tag{26}$$

Equations (23) to (25) may be considered to be the principal result of the present considerations. We read from this result in particular that, insofar as the location of the shear center relative to the elastic centroidal axis of the beam is concerned, this location depends on the distribution over the cross section of <u>both</u> axial normal stress stiffness and transverse shear stress stiffness. At the same time this location comes out to be independent of the plate twisting stiffness coefficient D_B of the cross section.

Two limiting special cases of interest are given by (i) the case of an "open" cross section, for which $t_1G_1 = 0$, and (ii) the case for which $A_1E_1 = 0$, as shown in Figure 3. We have, for the open cross section case that $D_C = 0$ as well as $D_{CQ} = 0$, in such a way that $D_C/D_{CQ} = -a$. Equation (22), in conjunction with (23) then reduces to the form

$$\text{(i)} \quad x_s = 2a + a\,\frac{E_1A_1 - E_2A_2}{E_1A_1 + E_2A_2}\,, \tag{27}$$

and therewith $a \leq x_s \leq 3a$, depending on the value of E_1A_1/E_2A_2.

When $A_1E_1 = 0$ then $D_R = 0$ as well as $D_{RM} = 0$, in such a way that $D_R/D_{RM} = -a$. We now have from (24) and (25)

$$\text{(ii)} \quad x_s = a - \frac{2aD_A}{D_A + D_C}\left(1 - \frac{t_2G_2 - t_1G_1}{t_2G_2 + t_1G_1}\right)\,, \tag{28}$$

and therewith $-3a < x_s < a$, depending on the values of c/a, t_1G_1, t_2G_2 and $t_{12}G_{12}$.

Direct asymptotic determination of x_s. We may obtain Eq. (22) directly by considering the differential equations (17a,b) for the limiting case $D_R = 0$ for which of the three boundary conditions in (18) only the condition $\theta(L) = 0$ remains relevant. In order to obtain the correct result for this limiting case, we must at the same time set $D_{MR} = 0$ in such a way that D_R/D_{RM} remains finite, consistent with the relations in Eqs. (7). We have then from (17a)

$$\psi = \frac{D_A - D_C}{D_A + D_C}\,\theta' - \left(\frac{D_R}{D_{RM}} - \frac{D_C}{D_{CQ}}\right)\frac{Q_o}{D_A + D_C}\,, \tag{29}$$

and from (17b)

$$\left(D_A+D_B+D_C - \frac{(D_A-D_C)^2}{D_A+D_C}\right)\theta' = T_o + \frac{D_C}{D_{CQ}} Q_o - \frac{D_A-D_C}{D_A+D_C}\left(\frac{D_R}{D_{RM}} - \frac{D_C}{D_{CQ}}\right)Q_o \quad , \tag{30}$$

and therewith

$$\frac{4D_AD_C+D_B(D_A+D_C)}{D_A+D_C}\theta = (z - L)\left[T_o - \frac{D_A-D_C}{D_A+D_C}\,\frac{D_R}{D_{RM}}\,Q_o + \frac{2D_A}{D_A+D_C}\,\frac{D_C}{D_{CQ}}\,Q_o\right] \quad , \tag{31}$$

with Eq. (31) once again implying the asymptotic result (22) for the quantity x_s.

It would be tempting to say that the assumption $D_R = 0$ means that the stress measure R becomes a reactive quantity, that is, must be determined through use of the equilibrium equation $R' = T_C - T_A$. However, what actually happens is that we now determine R through use of the constitutive equation (16d) (with $D_R = 0$ and $D_R/D_{RM} \neq 0$), with subsequent determination of a z-independent expression for ψ from the equilibrium equation involving R'.

A Remark on Generalizations. It is evident that an analogous analysis, involving six standard force and moment equilibrium equations, in conjunction with a seventh equilibrium equation of the form (3d) can readily be accomplished for a general four-corner cross section as depicted in Figure 4. What is not readily seen is how to establish rationally a seven one-dimensional equilibrium equation beam theory for a general n-corner cross section when $5 \leq n$ (as well as for homogeneous cross sections), over and above an approximate formulation via use of the Principle of Minimum Potential Energy, in conjunction with a suitable four parameter family of expressions (involving St. Venant's warping function) for the cross sectional distribution of axial displacement, analogous to what has been done in [4].

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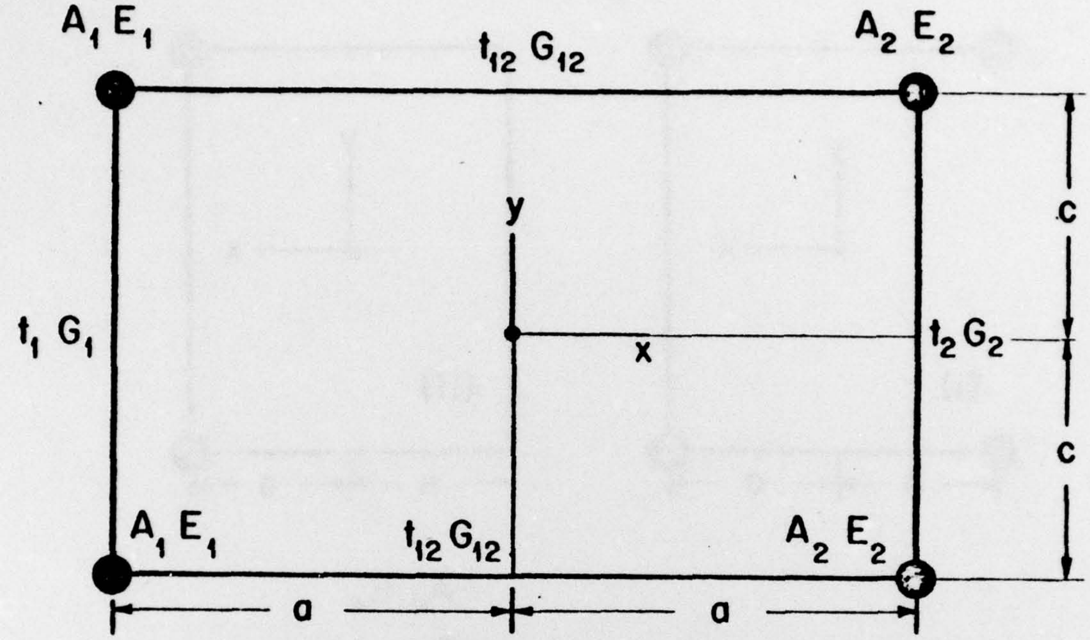
FIGURE 1



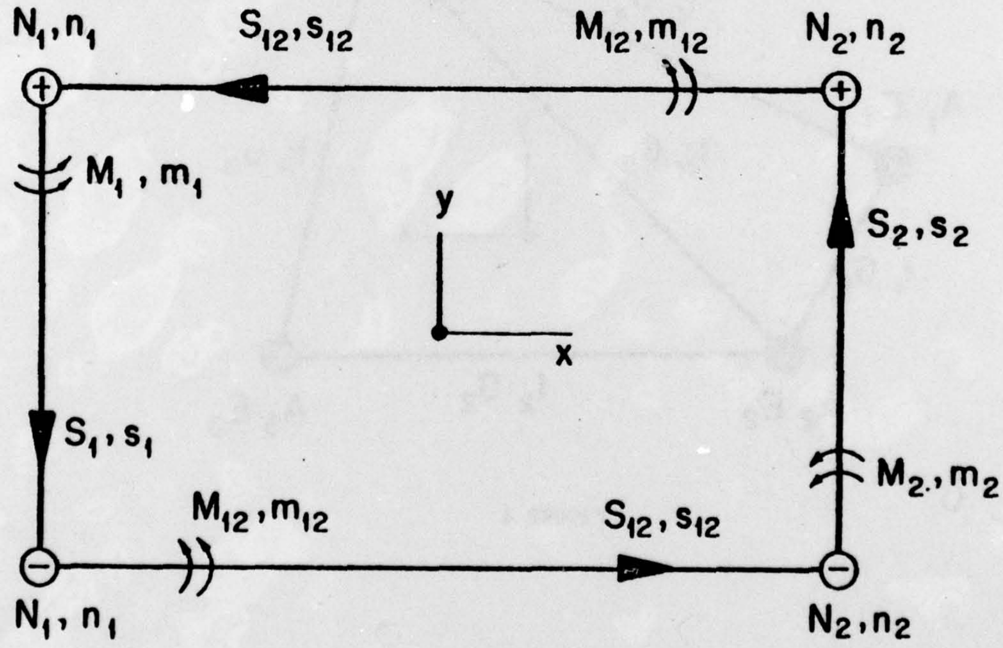
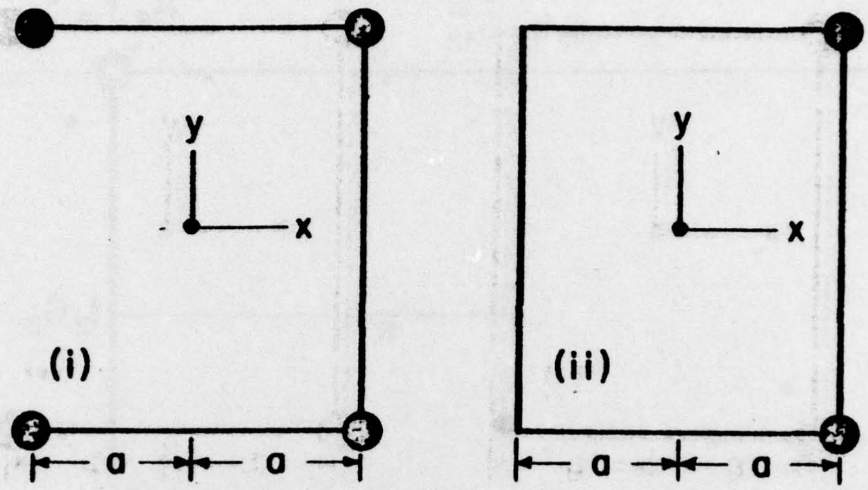


FIGURE 2

FIGURE 3



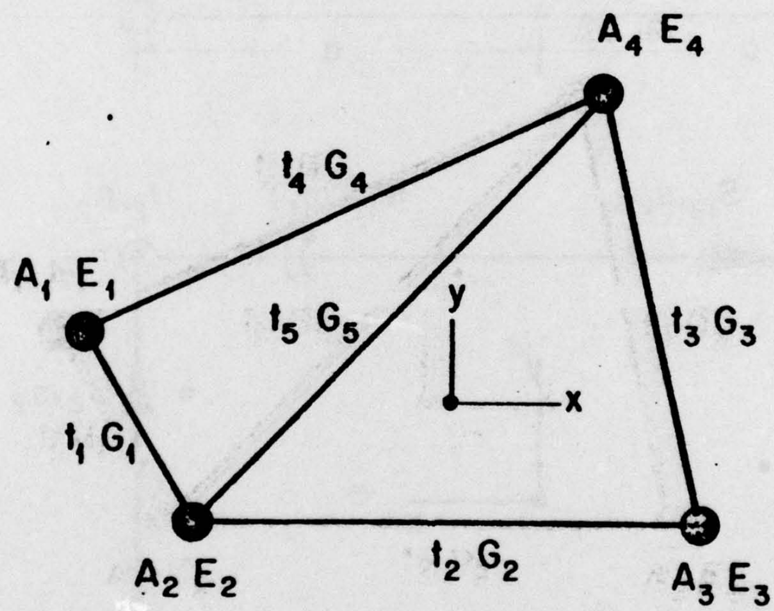


FIGURE 4

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Higher-order one-dimensional beam theory, effect of warping of cross section, shear center location as affected by cross sectional distribution of elastic moduli.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

One-dimensional beam equations are derived for a rectangular-cross section beam consisting of shear webs and corner stringers with one cross-sectional axis of symmetry. It is shown that a systematic statement of this problem involves one more equation than elementary theory, with this additional equation involving the concept of cross-sectional warping and bi-moment. As an application of the general results of the paper, new conclusions are deduced concerning the location of the center of shear of the beam.

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